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Assessing the variability in direct shear testing interpretation for sets of natural fractures

Franco, M.A., Potter, J. and Meyer, B.
Geotechnical Center of Excellence at the University of Arizona, Tucson, Arizona, USA

ABSTRACT: A direct shear test is a standard geotechnical laboratory test used to estimate the shear strength of rock fractures. This paper builds on previous work conducted by the authors to assess the extent of variation in shear strength parameters resulting from different approaches to interpreting direct shear test data. This interpretation involves selecting shear stresses from a shear stress versus shear displacement plot and utilizing them in the Mohr-Coulomb constitutive model to calculate friction angle and cohesion. The authors navigate through a series of direct shear results on fractures in the same rock type. Furthermore, a Python script was developed to automate the interpretation process, offering analysts an alternative tool. A variation up to 22.9 degrees in friction angle was observed and the effects of area and dilation corrections were analyzed.

1. INTRODUCTION

Direct shear tests (DST) are geotechnical tests designed to assess the shear strength and deformation properties of soils and rock discontinuities (MacDonald et. al. 2023). This test can be conducted either in a controlled laboratory setting using core specimens or in the field under in-situ conditions (Sanei et al. 2015).

A DST involves placing a core sample containing a plane of interest, referred to as a 'discontinuity,' onto a shear holder. The top part is displaced in relation to the bottom by applying a force parallel to the discontinuity (shear force) while maintaining a constant force perpendicular to the discontinuity (normal force). The resulting shear forces and corresponding shear displacements are measured and recorded. These forces are converted to stresses by dividing by the surface contact area, resulting in a shear stress vs. shear displacement trace (ASTM 2016). The test is generally performed three to five times on the same or similar samples with increasing applied normal loads. Figure 1 shows direct shear traces of a quartz monzonite fracture. The resulting data allows the interpreter to determine normal and shear stiffness, peak and residual shear strengths, and dilation angles (MacDonald et al. 2023).

Practitioners convert this information to shear strength by selecting two points along each shear stress versus displacement trace, which correspond to peak and residual (or ultimate) shear stress. The peak shear stress generally represents the maximum stress reached during the test, while the residual shear stress is associated with the point at which the shear stress remains essential constant with increasing shear displacement (Hencher and Richards 2015). Figure 2 shows an idealized shear stress versus displacement curve. Once the peak shear stress and residual shear stress are determined for each applied normal force, the values are matched to the corresponding normal stresses and used to estimate a failure envelope and associated friction angle and cohesion.

While selecting a peak shear stress value is relatively straightforward, determining an appropriate residual shear stress value can be a complicated and subjective process in cases where the behavior of the post-peak section of the curve fluctuates (Figure 1). The most widely used and applied guidelines, such as the American Society of Testing Materials (ASTM) Standard Test Method for Performing Laboratory Direct Shear Strength Tests of Rock Specimens, Designation: 5607 (ASTM 2016) and the ISRM Suggested Method (ISRM SM) for Laboratory Determination of the Shear Strength of Rock Joints (Muralha et al., 2014), lack detailed guidance on precisely how these points should be chosen or what constitutes good practice in this regard.

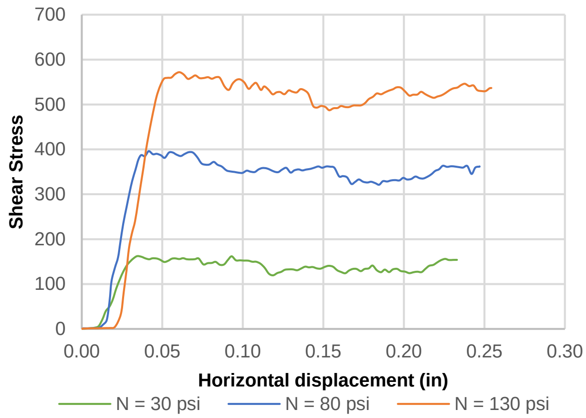

Fig 1. Direct shear traces for open quartz monzonite fracture (study sample obtained from University of Arizona Rock Mechanics Laboratory).

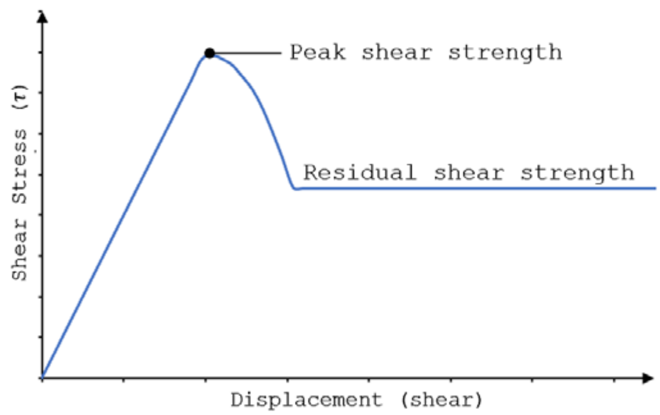

Fig 2. Idealized shear stress versus shear displacement trace, with points defining peak and residual shear strength identified. Modified from ASTM (2016).

The lack of guidance on interpreting direct shear testing meaningfully has been well documented. For instance, (Hencher and Richards 1989) and (Hencher 1995) claim that the older version of the ISRM Suggested Method published in 1981 and the CANMET report (Gyenge and Herget 1977) provide little advice on interpretation and are seldom rigorously followed in practice. However, it was acknowledged that these documents emphasize the significance of careful monitoring of displacement data as well as loads (Hencher 1995). Similarly, Hencher and Richards (2015) argue that published guidelines and rock mechanics textbooks often overlook the complexities of test data analysis, particularly when applied to large-scale projects.

The pressing need for comprehensive guidelines in the field is due to the extreme variability of shear strengths for rock discontinuities. Previous work has shown a variation in friction angle of approximately nine degrees when employing various approaches to select the residual shear stress for a single quartz monzonite sample (Franco et. al, 2023). Variations of up to 12.5 degrees in saw-cut samples tested in different laboratories have been documented by Nicholson (1994).

This paper summarizes data analysis and comparison of 16 DST results of natural fractures on granite samples based on the methodology proposed by Franco, et. al, (2023). Figure 3 shows three samples (top and bottom surfaces) that were included in the analysis. The objective of this work is to document how shear strength parameters can vary for natural discontinuities when different approaches to the selection of shear stress-normal stress pairs are utilized.

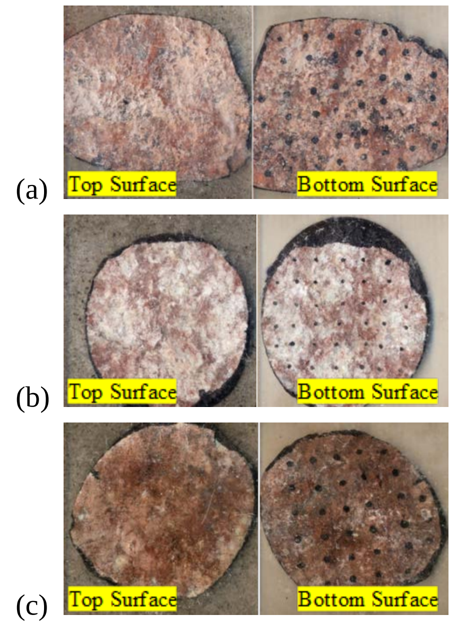

Fig 3. Pre-test pictures of three samples used in the analysis.

2. METHODOLOGY

2.1. Testing Procedures

A DST can be completed by employing one of two procedures: a single-stage DST or a multi-stage DST. In a single-stage DST, three to five samples from the same discontinuity, with approximately similar characteristics, are tested under increasing normal forces. Each surface is exposed to one normal and sheared only once (Muralha et al. 2014).

Multi-stage DST procedures are applied in cases where sample availability is limited. Multi-stage tests typically involve repositioning the sample once the practitioner determines that a residual value has been met for a given normal force. The sample is then reset to zero displacement, at which point the normal force is increased. In a DST without repositioning, the sample is sheared to a certain point, and the normal force is increased before continuing the test. The test continues from that point at a higher normal load (Muralha et al. 2014).

It is worth noting that the multi-stage DST procedure has faced criticism for potentially resulting in reduced peak shear strength parameters due to surface polishing (MacDonald, et. al, 2023). The values utilized for this study were collected utilizing multi-stage testing. However, this study primarily focuses on post-peak or residual values, for which multi-stage is less of a concern.

In a multi-stage DST analysis, a key outcome is the relationship between shear stress and shear displacement under three to five distinct normal stresses applied to the sample. This relationship is visualized in a shear stress vs. shear displacement plot, which typically contains three to five traces. These traces are used to estimate a failure envelope, which is crucial for understanding potential failure planes under different stress conditions.

To construct the failure envelope, a shear stress value is selected from each trace and pairs it with the corresponding normal stress. These pairs are then plotted in a shear stress vs. normal stress plane, enabling the derivation of a linear or non-linear relationship that describes the shear strength of the rock. In a linear relationship, this shear strength is given in terms of friction angle and cohesion.

2.2. *Quantifying Variability*

Variability is introduced during the interpretation state through the selection of shear stress values representing peak or residual strengths. These selections result in a failure envelope and, consequently, varying friction angle and cohesion values. To explore these variations, a Python code was developed to systematically select shear stresses from the post-peak region of a single set of traces using two approaches proposed by Franco, et. al, 2023, referred to here as the Displacement Approach and the Permutations Approach.

The Displacement Approach selects shear stresses at fixed increments of shear displacement and calculates the associated shear strength at each increment. The traces are divided into equal increments of displacement, and friction angle and cohesion are derived for each section. Figure 4 illustrates the Displacement Approach with a fixed increment of 0.254 cm (0.1 inch) applied to one of the datasets. The increments are represented with black dashed lines, and different dot colors represent the shear stresses that will be used to derive a linear failure envelope.

The Permutations Approach considers all possible arrangements of shear stresses from a single dataset by iterating through all possible combinations of shear stress values for each trace. Each unique combination of fixed and varied shear stress values represents a permutation. For example, in a scenario with three traces labeled A, B, and C, the approach is applied by fixing the first shear stress value from trace A in the post-peak region, iterating through all trace B and trace C values, and calculating the shear strength for each iteration. The same is done for every point on trace A, and then repeated for every combination of fixed shear stress values from traces B and C, with varying shear stress values for the non-fixed points. Figure 5 shows one example of one permutation in a dataset.

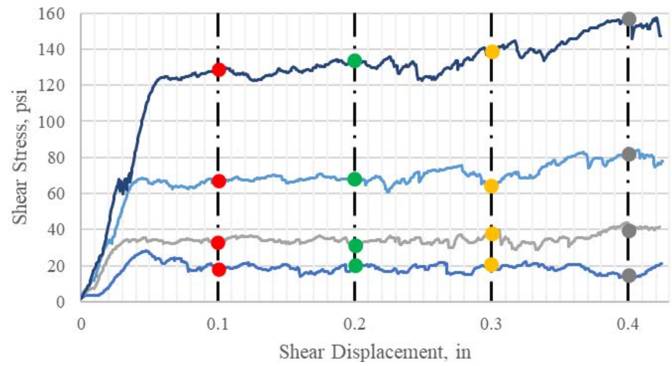

Fig 4. Displacement Approach set to 0.254 cm steps in displacement.

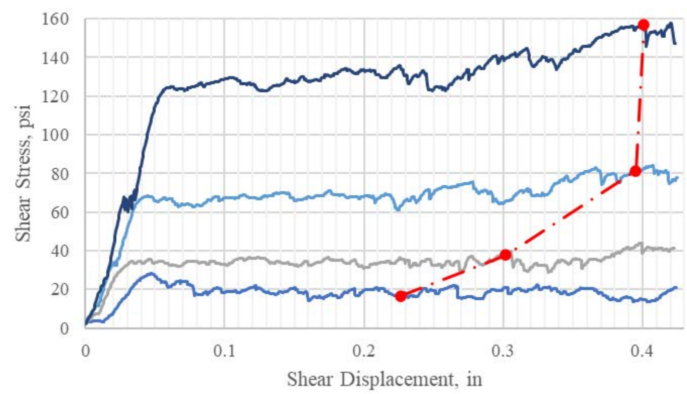

Fig 5. Permutations Approach displaying one random combination.

Both the Displacement and Permutations Approaches were applied to 16 DST results of discontinuities in granite samples from a mine site in Arizona. The samples represent fractures in a Precambrian granitic intrusive rock and were selected to be as homogeneous as possible in terms of roughness and mineralization characteristics.

The input parameters of the code include horizontal and vertical displacements, horizontal and vertical forces, and the area of the specimen, which is either an ellipse or closely resembles one, along with its major and minor axis dimensions. The horizontal and vertical forces are converted to stresses by dividing them by the contact area between the two surfaces of the discontinuity. As the two surfaces undergo shearing, the contact area gradually decreases, leading to an increase in both normal and shear stresses. To accommodate this variation in area, the equation proposed by Hencher and Richards (1989) is utilized:

$$A = \pi ab - \frac{ub\sqrt{4a^2-u^2}}{2a} - 2ab\sin^{-1}\frac{u}{2a} \qquad (1)$$

Where A is the overlapped area, u is the relative horizontal displacement, a is the major axis and b is the minor axis.

The calculated stresses are then corrected for the effects of dilation or compression that the discontinuity undergoes in the shearing process. Dilation refers to the tendency of the discontinuity to expand or dilate perpendicular to the direction of shearing. When a shear stress is applied to the discontinuity, the rock blocks on either side of the discontinuity begin to slide past each other. As this sliding occurs, the roughness and asperities on the discontinuity surfaces interact, causing the discontinuity to dilate. This correction is done by following the equations proposed by Hencher and Richards (1989):

$$i = \tan^{-1}\frac{dv}{dh} \quad (2)$$

$$\tau_i = (\tau \cos i - \sigma \sin i)\cos i \quad (3)$$

$$\sigma_i = (\tau \sin i + \sigma \cos i)\cos i \quad (4)$$

Where *i* is the dilation angle, *dv* is the difference between vertical displacements, *dh* is the difference between horizontal displacements, τ is the shear stress, σ is the normal stress, and the subscript *i* denotes corrected stresses.

The variation in horizontal displacement (*dh*) is calculated on a sample basis, computed as the average of the differences in horizontal displacement between each row and the subsequent row (0.001524 cm or 0.0006 inches in most cases). The variation in vertical displacement (*dv*) is calculated for each data entry in the dataset.

Once all the shear and normal stresses have been corrected for dilation, one shear stress per trace is selected according to the Displacement Approach or the Permutations Approach, coupled with its corresponding normal stress, and a linear regression is conducted. This linear regression follows the Mohr-Coulomb strength criterion (Hoek, 2023):

$$\tau = c + \sigma \tan \varphi \quad (5)$$

Where τ is the shear stress, *c* is the intercept with the y-axis, σ is the normal stress and φ is the friction angle.

In this context, the process of selecting shear stresses for all the traces contained in a DST and pairing them with appropriate normal stresses is termed a combination. The number of combinations can vary depending on the approach used, and this number dictates how many shear strength parameters (such as friction angle and cohesion) are calculated for a given method. For example, when the Displacement Approach is utilized, and the increment is set to be 0.0254 cm (0.01 inches), a total of 38 combinations are obtained from a single dataset. With an increment jump of 0.381 cm (0.15 in), three combinations are obtained. When the Permutations Approach is utilized, the number of combinations varies from thousands to millions, depending on the distance at which the peak shear stress was reached.

The number of combinations obtained makes it possible to create a range in which shear strength parameters could fall. This range of variability is a crucial aspect of this analysis. By repeating the procedure described before for all 16 samples, a comprehensive range of variability is established for the fractures analyzed. For comparison purposes, the same analysis is repeated without including area and dilatancy corrections. Additionally, mean friction angles resulting from the application of the script are compared to the friction angle values reported by the laboratory that provided the data for this analysis.

3. RESULTS

3.1. Individual sample analysis

This section presents the individual results of applying the Python script to the samples. To facilitate the visualization of the obtained values, only eight samples, which were randomly chosen, are displayed. The outcomes are maximum and minimum friction angle values, the difference between them, the mean friction angle, the standard deviation, and the coefficient of variation. Results from the Displacement Approach are presented in Table 1 and the Permutations Approach in Table 2. Finally, this section includes a comparison between the average friction angle results obtained for each sample using both the Displacement and Permutations Approaches, with the friction angles reported by the laboratory that provided the data. Table 3 displays the results.

Table 1. Maximum, minimum, difference, mean, standard deviation, and coefficient of variation (CV) of obtained friction angles for the Displacement Approach at 0.025 in.

Displacement Approach, degrees						
Sample #	Max	Min	Diff	Mean	STD	CV%
1	29.5	21.6	7.9	27.2	2.2	8.1
2	30.6	17.6	13.0	26.3	3.4	13.1
3	35.8	21.9	13.9	30.0	4.1	13.6
4	31.5	17.2	14.3	26.8	3.6	13.5
5	34.3	26.9	7.4	32.4	1.7	5.2
6	30.8	24.1	6.7	26.1	1.8	6.9
7	33.4	25.3	8.2	30.8	1.5	4.9
8	31.0	21.5	9.5	28.2	1.9	6.7

Table 2. Maximum, minimum, difference, mean, standard deviation, and coefficient of variation (CV) of obtained friction angles for the Permutations Approach.

Permutations Approach, degrees						
Sample #	Max	Min	Diff	Mean	STD	CV%
1	28.9	28.6	0.4	28.8	0.1	0.2
2	31.3	26.6	4.7	29.0	0.9	3.3
3	38.1	25.2	12.9	26.9	5.4	20.0
4	31.9	28.7	3.2	30.7	0.8	2.7

Permutations Approach, degrees						
Sample #	Max	Min	Diff	Mean	STD	CV%
5	34.4	27.8	6.6	31.6	1.4	4.4
6	29.5	28.4	1.0	29.0	0.3	0.9
7	31.3	30.3	1.0	30.7	0.1	0.4
8	31.8	24.9	6.9	27.3	1.4	5.2

Table 3. Mean friction angles comparison.

Sample #	Permutations Approach, degrees	Displacement Approach, degrees	Lab results, degrees
1	28.8	27.2	25.7
2	29.0	26.3	29.2
3	26.9	30.0	31.5
4	30.7	26.8	26.9
5	31.6	32.4	31.1
6	29.0	26.1	26.2
7	30.7	30.8	30.3
8	27.3	28.2	29.9

For 75% of the samples analyzed, the mean friction angle values calculated by the Python script were lower than the friction angles reported by the lab. Specifically, the Permutations Approach yielded lower friction angle values in 25% of cases, while the Displacement Approach resulted in lower friction angle values in 50% of the cases.

The difference between the mean friction angles obtained from the Permutations Approach and the laboratory results is smaller than the difference between the Displacement Approach and the laboratory results. When considering the total difference between both approaches, it is higher than the difference between either approach and the laboratory results.

3.2. *Combined sample analysis*

The samples underwent collective analysis to extract the range of variation, defined as the difference between the maximum and minimum friction angles, the mean friction angle, the standard deviation, and the coefficient of variation. This process was repeated twice more: once without applying Equation (1) but maintaining the gross area constant as the displacement progressed, and once without including Equations (2), (3), and (4), which account for the effects of surface dilation or contraction during the test. The findings are detailed in Table 4 for the Displacement Approach and Table 5 for the Permutations Approach.

4. DISCUSSION

4.1. *Friction Angle Trends in Displacement and Permutations Approaches*

When considering the calculated friction angle for all samples collectively, a maximum difference of 20.5 degrees is observed for the Displacement Approach and 22.9 degrees for the Permutations Approach. This discrepancy is logical as the Displacement Approach is a subset of the Permutations Approach. Notably, the largest difference in friction angles was observed for Sample # 4 in the Displacement Approach (14.3 degrees) and Sample #3 in the Permutations Approach (12.9 degrees).

Table 4. Displacement Approach results.

	Displacement Approach (DA)	DA (no area correction)	DA (no dilation correction)
Range, degrees	20.46	20.46	20.73
Mean Friction Angle, degrees	28.76	28.76	28.77
Standard Deviation, degrees	3.40	3.40	3.42
CV%	11.82	11.82	11.88

Table 5. Permutations Approach results.

	Permutations Approach (PA)	PA (no area correction)	PA (no dilation correction)
Range, degrees	22.91	22.39	22.94
Mean Friction Angle, degrees	28.69	28.51	28.73
Standard Deviation, degrees	3.24	3.22	3.32
CV%	11.29	11.29	11.56

4.2. *The Role of Area and Dilation Corrections*

When considering all the friction angles from all samples collectively, a maximum difference of 20.5 degrees is observed for the Displacement Approach and 22.9 degrees for the Permutations Approach. This discrepancy is logical as the Displacement Approach is a subset of the Permutations Approach.

It is common to apply corrections to the shear and normal stresses recorded during a DST that account for dilation (Equations 2-4) and diminishing area (Equation 1) during testing. To quantify the impact of these corrections, the shear strengths were calculated for the sample set before and after the corrections were applied. The results indicate that the corrections do not significantly alter the mean friction angle obtained for either approach (Tables 4 and 5). The dilation correction raises the mean by only 0.01 degrees for the Displacement Approach and by 0.8 degrees for the Permutations Approach. Similarly, the area correction maintains the mean for the Displacement

Approach, at least to the first two significant figures, while reducing the mean by 0.2 degrees for the Permutations Approach.

As initially assumed, neglecting the area correction leads to lower friction angles because the calculated stresses are smaller compared to when they are divided by a smaller contact area (which occurs as shearing progresses). Conversely, omitting the dilation correction results in slightly higher friction angles because it accounts for the work done when the asperities dilate. This suggests that, for most samples and at most points, dilatancy rather than contraction predominates, likely contributing to the slightly elevated results observed.

4.3. Comparison with Laboratory Reports

This comparison demonstrates that, overall, the average results from the Displacement and Permutations Approaches closely align with those reported by the laboratory. The maximum difference observed was 4.9 degrees for the Displacement Approach and 4.5 degrees for the Permutations Approach. In contrast, the smallest difference was 0.1 degrees for the Displacement Approach and no difference for the Permutations Approach.

Comparing the means, the differences are more pronounced between the Displacement Approach and the laboratory-reported results than between the mean obtained by the Permutations Approach and the laboratory results. The Permutations Approach tends to overestimate the friction angles, possibly because it considers more combinations than an analyst typically would. In contrast, the Displacement Approach has an equal likelihood of overestimating or underestimating the friction angles.

5. CONCLUSIONS

The goal of the work presented here is to document the potential variability in shear strength values that may result from differing approaches to the interpretation of direct shear test results. Two approaches to selecting shear stress values have been proposed here. These approaches were developed as tools to quantify the possible range in residual shear strengths for a given sample or sample set. Applying the Permutations Approach to 16 samples from the same granitic rock mass resulted in a 22.9-degree range in possible friction angles. The Displacement Approach resulted in a 20.1-degree range in possible friction angles. These variations can significantly impact a design, highlighting the critical importance of addressing and understanding the sources of variability.

This study focused on quantifying the variability that may be introduced by different interpretation techniques. However, it is important to consider that many other factors may contribute to variability in direct shear test results beyond the practitioner's selection of points in a shear stress vs. shear displacement curve. The application (or lack thereof) of common correction factors represents two additional potential sources of variability that were explored for this study, specifically area and dilation correction.

Correcting for the diminishing contact area that results as shearing progresses increases calculated friction angles when compared with values that do not apply an area correction. In contrast, applying dilation correction results in slightly lower mean friction angles. For the study set tested here, these corrections resulted in minimal changes to the mean friction angle. This finding is consistent with existing literature, but further research is needed to assess the impact of this finding.

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